Name: Key Date: _____

A2CC Q3T3 Review

THIS REVIEW IS NOT COMPREHENSIVE. BE SURE TO STUDY YOUR NOTES, HOMEWORK ASSIGNMENTS, AND OLD TESTS AS WELL!

1. The exponential function $y=16\left(2^{x}\right)$ could be rewritten as

(1) $y=32^{x}$ (3) $y=2^{x+4}$

(2) $y=2^{5x}$ (4) $y=2^{x^3}$

$16=2^4$

$y=2^4(2^x)=2^{4+x}$

2. Which of the following would give the same result as $\left(\sqrt{\sqrt[3]{2}}\right)^4$?

$=\left(\sqrt{2^{1/3}}\right)^4=\left(\left(2^{1/3}\right)^{1/2}\right)^4=2^{2/3}$

$\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4}{1}=\frac{4}{6}=\frac{2}{3}$

$=\sqrt[3]{2^2}=\sqrt[3]{4}$

(1) $\sqrt[5]{8}$ (3) $\sqrt{2}$

(2) $\sqrt[4]{8}$ (4) $\sqrt[3]{4}$

3. For the function $f(x)=5(2)^{x}+7$, which of the following represents its y-intercept?

$x=0$

(1) 7 (3) 12

(2) 5 (4) 17

$f(0)=5(2)^0+7$

$=5(1)+7$

$=5+7=12$

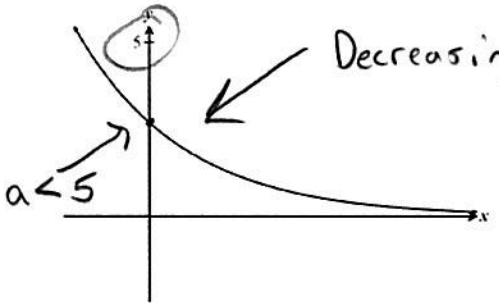
4. Which of the following could be the equation of the graph shown below?

(1) $y=10(0.5)^{x}$

(2) $y=3(0.75)^{x}$

(3) $y=4(1.25)^{x}$

(4) $y=5(2.2)^{x}$



Decreasing ∴ base is less than 1

$a<5$

$y=ab^x$

5. Selected values of an exponential function of the form $y = a(b)^x$ are shown below. To the nearest hundredth, which of the following represents the value of b?

(1) 1.13 (3) 1.78

(2) 1.55 (4) 2.03

x	2	5	7	10	15
y	71	102	130	187	341

Use any 2 points

(2,71) (5,102)

$\frac{102}{71} = \frac{ab^5}{ab^2} \rightarrow \frac{102}{71} = b^3$ $\sqrt[3]{\frac{102}{71}} = b$

6. In terms of the unknown constant a, which of the following values of x solves: $(27)^{2x-a} = \left(\frac{1}{9}\right)^{x+3a}$?

(1) $-\frac{a}{2}$ (3) $-\frac{3}{8}a$

(2) $\frac{4}{3}a$ (4) $4a$

$27 = 3^3$ $\frac{1}{9} = 3^{-2}$

$(3^3)^{2x-a} = (3^{-2})^{x+3a}$

$6x - 3a = -2x - 6a$

$+2x \quad +3a \qquad +2x \quad +3a$

$\frac{8x}{8} = \frac{-3a}{8}$ $\rightarrow x = \frac{-3}{8}a$

7. A population of fruit flies is increasing at a rate of 22.5% per hour. If the population had an original size of 10 flies, then which of the following is its size after one day?

(1) 798 (3) 1122

(2) 935 (4) 1304

.225

24 hours

$y = 10(1+.225)^{24}$

$y = 10(1.225)^{24}$

$y = 1303.97$

8. The water level in a draining reservoir is changing such that the depth of water decreases by 7.5% per hour. If the water starts at a depth of 45 feet, then which of the following functions properly models the depth, d, as a function of time, t, in hours since it started draining?

(1) $d = 45(.075)^t$ (3) $d = 45(7.5)^t$

(2) $d = 45(.925)^t$ (4) $d = 45(92.5)^t$

.075

$d = 45(1 - .075)^t$

$d = 45(.925)^t$

9. The temperature of a cooling liquid in a room held at a constant 75 degrees Fahrenheit can be described by the equation $F(t)=132(.97)^{t}+75$, where F is the Fahrenheit temperature and t is the amount of time it has been cooling, in minutes. Which of the following was the original temperature of the liquid when it began cooling?

$t=0$

(1) 75 (3) 203

(2) 132 (4) 207

$F(0)=132(.97)^{0}+75$

$=132(1)+75$

$=132+75=207$

10. If a population grows at a constant rate of 2.8% per year, then by what percent will it grow over the next 10 years?

.028

(1) 17% (3) 32%

(2) 28% (4) 39%

$(1+.028)^{10}=1.318=(1+.318)$

↑ growth rate $\approx 32\%$

11. Which of the following is closest to the value of $\log_4(40)$?

(1) 1.8 (3) 2.7

(2) 2.3 (4) 3.5

Use calculator

Math | Alpha | Math

12. If $b>0$ then $\log_b\left(\frac{1}{b^3}\right)$ is equal to

(1) $\frac{1}{3}$ (3) 3

(2) $\frac{b}{3}$ (4) -3

$\log_b 1-\log_b b^3$

$0-3\log_b b$

$0-3(1)$ → $0-3=-3$

13. Given the function $f(x)=\log_2(2x-8)$, which of the following values of x is *not* in the domain of the function?

(1) $x=5$ (3) $x=8$

(2) $x=2$ (4) $x=20$

$2x-8>0$

$+8 \quad +8$

$\frac{2x}{2}>\frac{8}{2}$

$x>4$

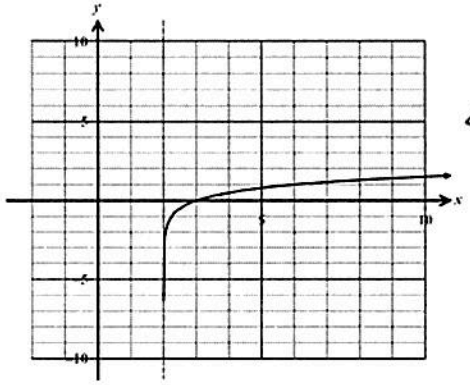
14. Which of the following equations is shown graphed on the grid below?

(1) $y = 2^x + 2$

(2) $y = 4^x - 2$

(3) $y = \log_4(x-2)$

(4) $y = \log_2(x+2)$



log shifted right 2 units

15. Which of the following is equivalent to $\log\left(\frac{x^2}{\sqrt{y}}\right)$?

(1) $(\log x)^2 - \sqrt{\log y}$

(2) $\frac{2\log x}{\sqrt{\log y}}$

(3) $\frac{2\log x - \log y}{2}$

(4) $\frac{4\log x - \log y}{2}$

$\log x^2 - \log \sqrt{y}$

$2\log x - \log y^{1/2}$

$2\log x - \frac{1}{2}\log y$

$2\log x - \frac{\log y}{2}$

$\frac{4\log x}{2} - \frac{\log y}{2}$

(*) $\sqrt{y} = y^{1/2}$

16. If $\log_b(5) = 1.2$ then $\log_b(125) = ?$

(1) 0.4

(2) 1.728

(3) 3.6

(4) 30

$125 = 5^3$

$\log_b(5^3) = 3\log_b 5 = 3(1.2) = 3.6$

17. If $5b^{x-3} = 7$ then $x =$

(1) $\frac{\log_b(7)}{5} + 3$

(2) $\frac{5b}{7} - 3$

(3) $3 + \log_b(1.4)$

(4) $3b^{7/5}$

$b^{x-3} = \frac{7}{5}$

$x - 3 = \log_b \frac{7}{5}$

$x - 3 = \log_b 1.4$

$+3 \quad +3$

$x = (\log_b 1.4) + 3$

18. If $f(x) = 50(0.92)^x + 75$ then which of the following values of x solves the equation $f(x) = 90$?

(1) 12.1

(2) 14.4

(3) 15.8

(4) 18.3

$90 = 50(.92)^x + 75$

$-75 \quad -75$

$\frac{15}{50} = \frac{50(.92)^x}{50}$

$\frac{3}{10} = (.92)^x \rightarrow x = \log_{.92} \frac{3}{10}$

Get t by itself

19. If $ae^{kt} - c = 0$ then which of the following is the value of t based on a, k, and c and the natural base e?

(1) $\frac{1}{k}\ln\left(\frac{c}{a}\right)$ (3) $\ln\left(\frac{c}{ak}\right)$

(2) $\frac{\ln(c)}{ak}$ (4) $\frac{ac}{ke}$

$ae^{kt} - c = 0$

$+c \quad +c$

$\frac{ae^{kt}}{a} = \frac{c}{a}$

$e^{kt} = \frac{c}{a}$

convert to ln form

$kt = \ln\left(\frac{c}{a}\right)$

$t = \frac{1}{k}\ln\left(\frac{c}{a}\right)$

20. If a liquid is cooling down according to the formula $y = 84e^{kt} + 55$ and at $t = 22$ the temperature is $y = 71$ then which of the following is the value of k to the nearest hundredth?

(1) −0.08 (3) 0.29

(2) −0.27 (4) 0.58

$71 = 84e^{22k} + 55$

$-55 \quad -55$

$\frac{16}{84} = \frac{84e^{22k}}{84}$

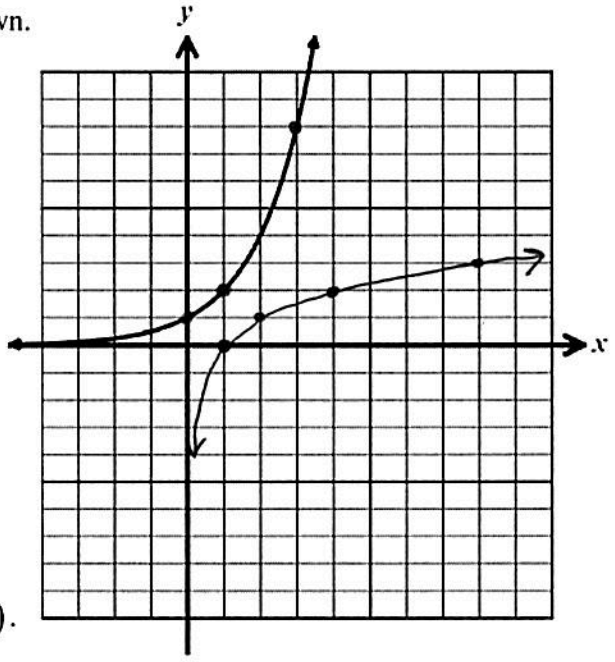
$\frac{16}{84} = e^{22k}$

convert to ln form

$22k = \ln\frac{16}{84}$

$k = \frac{\ln\left(\frac{16}{84}\right)}{22}$

21. On the grid shown below, the graph of $f(x) = 2^x$ is shown.



(a) On the same graph grid, create an accurate sketch of this function's inverse, $f^{-1}(x)$.

$f(x)$	$f^{-1}(x)$
(0, 1)	(1, 0)
(1, 2)	(2, 1)
(2, 4)	(4, 2)
(3, 8)	(8, 3)

$f(x) = 2^x$

$y = 2^x$

$f^{-1}(x) = 2^y$

(b) State the equation of $f^{-1}(x)$.

$x = 2^y$ or $y = \log_2 x$

(c) State the domain and range of both $f(x)$ and $f^{-1}(x)$.

$f(x)$

Domain: All Real #s

Range: $y > 0$

$f^{-1}(x)$

Domain: $x > 0$

Range: All Real #s

22. An object is slowing down such that its speed is decreasing exponentially. If after 2 seconds it is traveling at 58 feet per second and after 5 seconds it is traveling at only 11 feet per second, then find an equation in the form $y = a(b)^x$ for the speed, y, as a function of the number of seconds, x, that have passed. Find the equation using algebraic techniques. Round both a and b to the nearest hundredth.

Points
$(2, 58)$ $(5, 11)$

$$\frac{11}{58} = \frac{ab^5}{ab^2}$$

$$\frac{11}{58} = b^3$$

$$\sqrt[3]{\frac{11}{58}} = b$$

$b = .57$

$$11 = a(.57)^5$$

$$\frac{11}{(.57)^5} = a$$

$a = 175.71$

$y = 175.71(.57)^x$

23. The expression $\left(\sqrt[3]{b}\right)^5\left(\frac{1}{b^2}\right)$ can be written as b^a in simplest form. Determine the value of a. Show how you arrived at your answer.

$$\left(\sqrt[3]{b}\right)^5 = b^{5/3}$$

$$b^{5/3} \cdot b^{-2} = b^{\frac{5}{3}-2}$$

$a = \frac{-1}{3}$

$$\frac{1}{b^2} = b^{-2}$$

24. If $g(x) = \left(\frac{1}{5}\right)^{2x+7} - 3$ then algebraically determine the solution to the equation $g(x) = 22$.

$$22 = \left(\frac{1}{5}\right)^{2x+7} - 3$$

$+3 \quad +3$

$$25 = \left(\frac{1}{5}\right)^{2x+7}$$

convert to log form

$$2x + 7 = \log_{1/5} 25$$

$-7 \quad -7$

$$\frac{2x}{2} = \frac{(\log_{1/5} 25) - 7}{2}$$

$$x = \frac{(\log_{1/5} 25) - 7}{2}$$

$x = \frac{-9}{2}$

25. For the logarithmic function $f(x) = \log_2(x-4)$, explain why $x = 0$ is not in its domain.

If $x = 0$ then $f(x) = \log_2(-4)$

There is no power that we can raise 2 to inorder to get a negative number.

Turn 40 into

26. For some base, b, it is known that $\log_b(5) = 1.28$ and $\log_b(2) = 0.55$. For the same base, determine the value of $\log_b(40)$. Explain how you found your answer.

40 → 2, 20 → 2, 10 → 2, 5

$40 = 2^3 \cdot 5$

$\log_b(2^3 \cdot 5) = \log_b 2^3 + \log_b 5$

$3\log_b 2 + \log_b 5$

$3(.55) + 1.28$

$= \boxed{2.93}$

27. If the population of Ashmore, Illinois is decreasing by 5.8% per year, then by what percent will it decrease in the next 5 years? Show how you arrived at your result. Round to the nearest tenth of a percent.

yearly decrease is $(1 - .058)$ or $(.942)$

In 5 years $(.942)^5 = (.7417)$ 74.17% left

$1 - .7417 = .25825 \rightarrow$ **25.8 % decrease**

28. A liquid with an initial temperature of 194 °F cools in a room whose temperature is held at 68 °F. The temperature of the liquid, T, as it cools can be modeled as a function of time, x, using:

$$T(x) = (T_i - T_r)e^{kx} + T_r$$

Where T_i is the initial temperature, T_r is the temperature of the room and k is the decay constant.

(a) If $T(15) = 102$ then find the value of k accurate to the nearest hundredth.

$\frac{102}{-68} = (194 - 68)e^{K(15)} + 68 - 68$

$\frac{34}{126} = \frac{(126)e^{15K}}{126}$

$\frac{34}{126} = e^{15K}$ → convert to ln form

$\frac{15K}{15} = \frac{\ln\left(\frac{34}{126}\right)}{15}$

$K = \frac{\ln\left(\frac{34}{126}\right)}{15}$ **$K = -.09$**

(b) How many minutes does the model predict it will take for the liquid to reach a temperature of 70 °F? Round to the nearest minute and show or explain how you arrived at your answer.

$\frac{70}{-68} = (194 - 68)e^{-.09x} + 68 - 68$

$\frac{2}{126} = \frac{126\,e^{-.09x}}{126}$

$\frac{2}{126} = e^{-.09x}$

$\frac{-.09x}{-.09} = \frac{\ln\left(\frac{2}{126}\right)}{-.09}$

$x = \frac{\ln\left(\frac{2}{126}\right)}{-.09} =$ **46 minutes**